Study of the behavior of reinforced concrete deep beams. Estimate of the ultimate shear capacity

Estudio del comportamiento de vigas de gran peralto de hormigón armado. Estimación de la capacidad resistente última

Gilberto Rodríguez Plasencia (Main Author) (Correspondant Author)
Departamento de Matemática Aplicada, Universidad de Ciego de Ávila, Cuba
Carretera a Morón km 9 ½, Ciego de Ávila, Cuba. CP 069450, Fono: +53-33-266211
E-mail: gilbertor@unica.cu

Jorge Douglas Bonilla Rocha
Departamento de Matemática Aplicada, Universidad de Ciego de Ávila, Cuba
E-mail: jorgedbr@unica.cu

Juan José Hernández Santana
Departamento de Ingeniería Civil, Universidad Central de Las Villas, Cuba
E-mail: jjhernandez@uclv.edu.cu

Lame Pudipedi
Ingeniería Civil, Universidad de Ciego de Ávila, Cuba
E-mail: lamzy@gmail.com

Manuscript Code: 605
Date of Acceptance/Reception: 23.03.2017/27.03.2015
DOI: 10.7764/RDLC.16.1.43

Abstract
In this paper is carried out a study to the behavior of reinforced concrete deep beams by numerical simulation, where the results of numerical analysis are calibrated and validated from existing experimental results in the international literature. In modelling is considered a bilinear model with Von Mises failure criteria for steel and Concrete Damage Plasticity for concrete, achieving an appropriate match between the numerical results against the experimental reference, a feature that allows the parametric study of the influence of geometric factors and reinforcement. Two formulas for estimating the ultimate shear strength of reinforced concrete deep beams under static load with predominance of shear strength as proposed, which are based on empirical models developed from the processing of a large and profuse database where experimental results of the numerical simulation are included. These formulations show adequate accuracy in predicting the ultimate shear strength in these structures compared with some methods implemented on existing codes ACI Comitee 318 (2011) and European Standard EN 1992-1-1 (2004).

Key words: Reinforced concrete deep beams, numerical models, no lineal regression, ultimate shear capacity.

Resumen
En este trabajo se realiza un estudio del comportamiento de las vigas de gran peralto de hormigón armado mediante simulación numérica, donde los resultados obtenidos del análisis numérico son calibrados y validados a partir de resultados experimentales existentes en la bibliografía internacional. En la modelación se considera un modelo bilineal con criterio de rotura de Von Mises para el acero y un Modelo de Daño Plástico para el hormigón, lográndose una adecuada correspondencia entre los resultados numéricos en relación con los experimentales de referencia, aspecto este que permite el estudio paramétrico de la influencia de factores geométricos y del refuerzo. Se proponen además dos formulaciones para la estimación de la capacidad resistente última en vigas de gran peralto de hormigón armado bajo carga estática con predominio del esfuerzo cortante, las cuales están basadas en modelos empíricos desarrollados a partir del procesamiento de una amplia y profusa base de datos experimentales donde se incluyen además los resultados de la simulación numérica. Estas formulaciones muestran una adecuada precisión en el pronóstico de la capacidad resistente en este tipo de estructuras en comparación con algunos métodos implementados en normativas de referencia como ACI Comitee 318 (2011) y European Standard EN 1992-1-1 (2004).

Palabras clave: Vigas de gran peralto de hormigón armado, modelos numéricos, regresión no lineal, capacidad resistente última.

Introduction and Description of the Problem

ACI Comitee 318 (2011) defines beams with clear span to effective depth ratios less than 5 as deep beams, in these structures several possible modes of failure of deep beams have been identified from physical tests but due to their geometrical dimensions shear strength appears to control their design. Despite of the large amount of research carried out over the last century, there is no agreed rational procedure to predict the shear strength of reinforced concrete deep beams. This is mainly because of the very complex mechanism associated with the shear failure of reinforced concrete beams.

In the international literature there is a strong interest in studying the behavior of reinforced concrete deep beams with predominance of shear, expressed by numerous experimental studies in recent decades, standing very referenced works such as experimental studies of Kong, Robins, & Cole (1970), Smith & Vantsiotis (1982), Tan et al. (1995), Kong & Rangan (1998), Cladera & Marí (2005), among others. Aspect that undoubtedly contributes significantly to the improvement of methods for estimating the shear capacity in this type of structures. It has further been observed that there are areas with sharp discontinuities or disruptions in the stress fields (region D) in such beams. In these elements is not valid the hypothesis of plane sections of classical bending theory, based on the traditional methods of design and test sections.

The researches to understand the behaviour of deep beams has a short history relative to that in slender beams. It was not until the 1960s that tests to evaluate the capacity of deep beams were carried out by de Paiva & Siess (1965) and Ramakrishnan & Ananthanarayana (1968). These tests showed that failure modes of deep beams were dependent on span-to-depth ratio (a/d) and shear failure of beams having ratio (a/d) less than 2.0 was always initiated owing to splitting failure of diagonal concrete struts. They concluded that the span-to-depth ratio is the most influential factor governing the behavior of deep beams, noting that this factor is among the most influential governing the behavior of such beams and can be taken as the standard for the differentiation of these structures. It must be noted that in this sense the ultimate shear capacity of deep beams is multivariate, depending mainly on many geometric parameters of the properties of materials, the amount and type of reinforcement placed.

The numerical simulation represents additional aid to the design and modelling of physical tests, may be obtained comparable results with actual trials. Are countless works where this tool is used in different fields of engineering, where we can highlight among them: in the work of Norambuena-Contreras, Castro-Fresno, Del Coz, & García (2011) is studied the reflective cracking in samples of asphalt by simulating a dynamic test, in composite structures are marked the studies of Bonilla, Mirambell, Larrúa, & Recarey (2012), Hernández, Bonilla, & Rodríguez (2014) and Bonilla, Bezerra, Larrúa, Mirambell, & Recarey (2015), and for to study reinforced concrete structures according Sagaseta & Vollum (2010) and Adhikary, Li, & Fujikake (2013) are applied modelling as a complement to the study of the behavior of deep beams by simulating actual tests.

Today with the advances made in software, multipurpose programs have been developed that allow the numerical simulation of structures based on Finite Element Method. This paper realized a parametric study of behavioral factors that influence the shear capacity in reinforced concrete deep beams, based on adequate calibration of numerical models which are validated in correspondence with experimental results presented in studies of Salamy, Kobayashi, & Unjoh (2005). The results of numerical modelling showed the reliability in predicting the behavior of deep beams in terms of shear capacity, deformation, crack propagation and failure mode. These numerical results are taken into account in the regression analysis to estimate shear capacity of these structures, marking a significant approximation to the actual tests and achieving better correlations with the evaluated parameters on numerical simulation.

Despite the extensive experimentation and modelling carried for decades, there are still differences in the structural design criterion of these structures which shows have certain "special features". Shear design provisions of deep beams in different codes of practice can be classified into two categories, one of which involves empirical formulae and the other is based on strut-and-tie models. Code provisions for deep beams were usually developed from limited test results. In a search of the international literature are evidenced different methods for estimating the ultimate shear strength of deep beams, highlighting procedures ACI Comitee 318 (2011), European Standard EN 1992-1-1 (2004) and the formulations of Tang & Tan (2004).

In this paper, is defended a return to the empirical expressions obtained by statistical methods, which largely outperforms all approaches discussed in its response to the experimental study. The proposed formulations also allow a simple evaluation of the influence of the main elements resistant in the deep beams based on the traditional approach of the shear cross-sections.

Nomenclature

b_w: width of the beam (m);
d: effective depth (m);
a: effective shear span (m);

f'_c: compressive concrete cylinder strength (MPa);
ρ_{h-w}: ratio of horizontal web reinforcement (%);
ρ_{v-w}: ratio of vertical web reinforcement (%);
ρ_s: ratio of principal reinforcement (%);
f_{y-med}: average yield strength of web reinforcement (MPa);
f_{y-s}: yield strength of principal reinforcement (MPa).

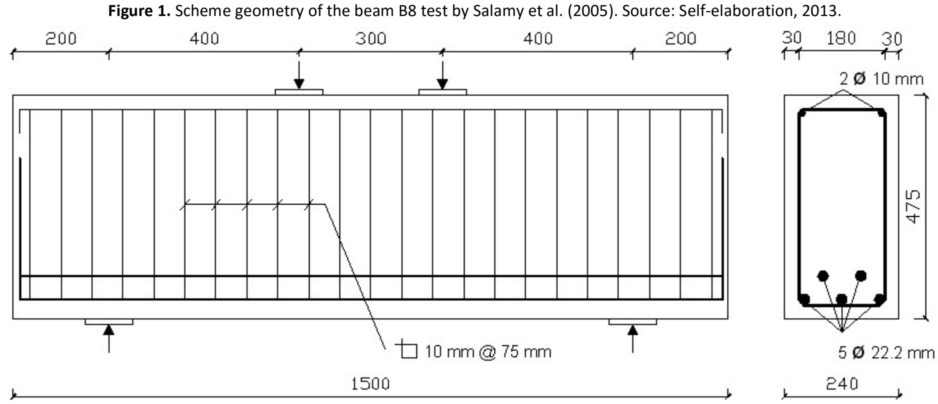
Methodology

Numerical analysis of specimens of deep beams

In this study all geometric and reinforcement elements are represented for proper numerical model assay of reinforced concrete deep beams, which simulates physical schemes of the beams test analyzed. To achieve this objective has been used the software ABAQUS, which is a general purpose software is based on the finite element method and has been widely used for numerical simulation of composite structures in studies of Ellobody & Young (2006) and Bonilla et al. (2012). These tools are much utilized given the large number of facilities provided to model three-dimensional geometries, nonlinearity of materials, including interfaces between elements and various boundary conditions.

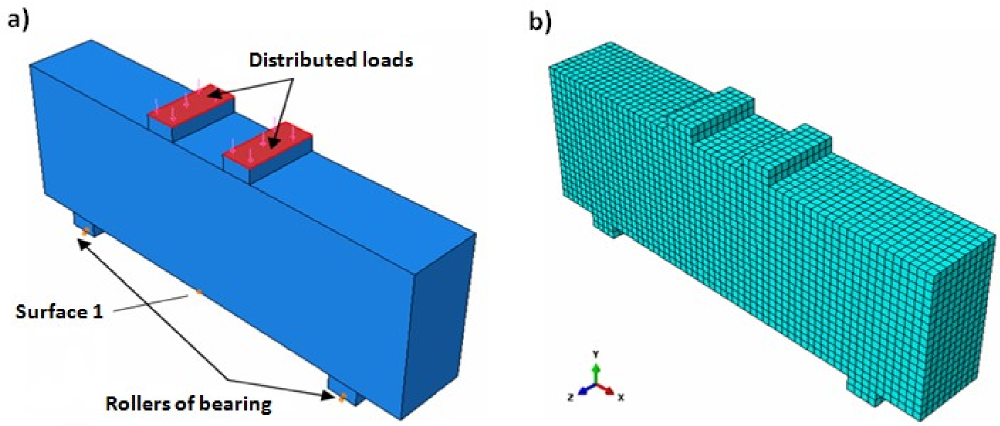
The specimen studied for calibration of the numerical simulation was B8 (Figure 1), where the compressive strength of concrete was 37.8 MPa, the yield stress of reinforcing steel has 376 MPa and a modulus of deformation equal to 210000 MPa.

Figure 1. Scheme geometry of the beam B8 test by Salamy et al. (2005). Source: Self-elaboration, 2013.



Boundary conditions of model

Figure 2. Schematic geometric model: a) Boundary conditions, b) Discrete model. Source: Self-elaboration, 2015.



The contact between the concrete beam and the plates is treated as rigid, due to the presence of high friction force, caused by large normal forces that prevent the movement at the interface.

The reinforcement bars in the concrete beam ensure high adhesion at the interface steel-concrete due to the corrugated that prevent sliding of the bars in the concrete, this justifies the use of the command *EMBEDDED ELEMENT implemented in ABAQUS and which is designed to model the behavior of reinforcing bars embedded in concrete. In this case the degrees of freedom of the nodes of the elements (truss) are constrained by the host (the concrete beam), so that the deep beams are deformed similarly which the steel truss reinforcing without sliding at the interfaces.

In the supports are restricted in axis Y and Z the displacements and are released in the X, thus simulating the support roller placed on the actual physical scheme of the test. At centre of the concrete beam represented as a partition face as shown on the Surface 1 (see Figure 2a) where the displacement is restricted in X and displacement in axis Y and Z are liberated to reach equilibrium of the numeric models given by support conditions.

Selection of finite element and meshing

Are conducted a study to selecting the type of finite element optimum that was capable to simulate the real physical behavior of deep beams. For this purpose, was discretized each geometry of the specimen with elements: C3D4, C3D6 and C3D8R. The best approximation with respect to the physical model is achieved with C3D8R elements for the discretization of the domain of the continuous medium of all solid bodies, in addition to these elements have a geometry that adapts very well to the modelled volumes, allowing a uniform meshing (see Figure 2b) in all models for the discretization of the reinforcing steel bars are used truss elements T3D2.

Different meshing on models was analyzed with respect to the precision and computational cost; was observed the favorable trend it is taking the value of ultimate shear capacity to the extent that the mesh is denser in the models and how that value gets closer better to the experimental value of reference, obtaining differences lower than 3% from the experimental value. It should be added that has sought the finite element mesh meets the aspect ratio of three-dimensional solids.

Application of the load

The load is applied on top of the plates at short intervals (see Figure 2a), where the size of such intervals was selected automatically by ABAQUS, based on the condition of numerical convergence. The loads are applied using the RIKS modified algorithm available in the program. The basis of this algorithm is Newton's method, which is generally used to predict nonlinear instability and collapse of a structure.

Concrete model

Concrete is modelled considering a plastic damage model developed by Lubliner, Oliver, Oller & Oñate (1989) and modified by Lee & Fenves (1998) available in ABAQUS library system and used for the numerical modelling of composite steel and concrete structures. The model considers the most important phenomena of concrete based on the theoretical principles of the model Mohr-Coulomb Modified and was created to study the effects of irreversible damage associated with failure mechanisms that occur in concrete. For the model calibration, must be introduced as discrete points, the behavior curves of compressive and tensile strengths for different resistance of concrete obtained from uniaxial tests.

In this model the deformation is decomposed in an elastic part and other plastic being expressed as:

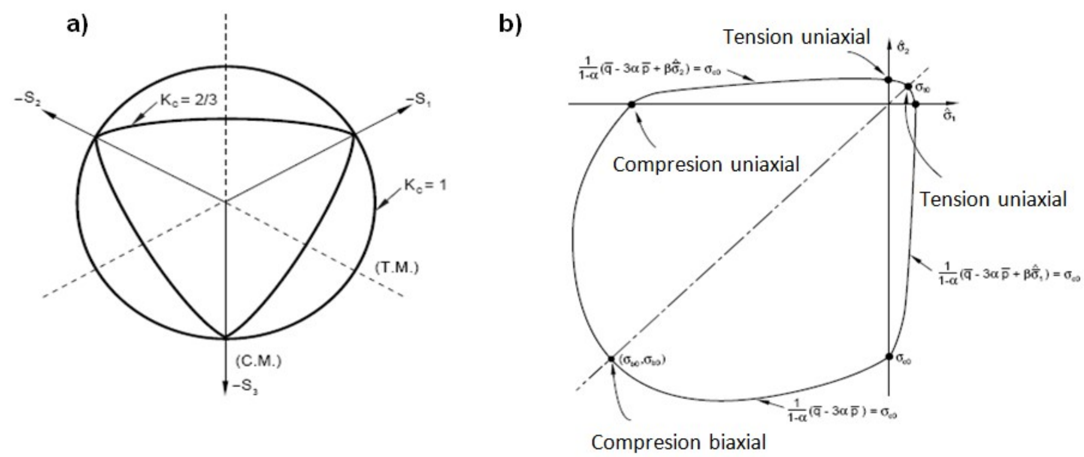
$$\dot{\varepsilon} = \dot{\varepsilon}^{el} + \dot{\varepsilon}^{pl} \quad (1)$$

where $\dot{\varepsilon}$ is total deformation, $\dot{\varepsilon}^{el}$ is the elastic deformation and $\dot{\varepsilon}^{pl}$ is the plastic deformation. The deformation strain relationship is governed by the expression:

$$\sigma = (1-d)D_0^{el} : (\varepsilon - \varepsilon^{pl}) = D^{el} : (\varepsilon - \varepsilon^{pl}) \quad (2)$$

where D_0^{el} is the elastic stiffness of the undamaged material, $D^{el} = (1-d)D_0^{el}$ is degraded elastic stiffness and d is variable stiffness degradation which takes values in a range between zero for no wound material and one for the totally corrupted material. Damage associated with the failure mechanisms of the concrete occurs as the result of a reduction in the elastic stiffness of the material.

Figure 3. Yielding surface in: a) The stresses deviator, b) Plane of stresses. Source: ABAQUS (2014).



The damage is considered in tension and compression and is characterized independently by two variables of hardening $\tilde{\varepsilon}_t^{pl}$ and $\tilde{\varepsilon}_c^{pl}$, which refer to equivalent plastic deformations in tension and compression, respectively. The evolution of hardening variables is provided by an expression of the form:

$$\tilde{\varepsilon}^{pl} = \begin{bmatrix} \tilde{\varepsilon}_t^{pl} \\ \tilde{\varepsilon}_c^{pl} \end{bmatrix}; \quad \dot{\tilde{\varepsilon}}^{pl} = h(\bar{\sigma}, \tilde{\varepsilon}^{pl}) \cdot \dot{\varepsilon}^{pl}, \quad (3)$$

The micro-cracks and the crushing of concrete are represented by the increasing values of the variables of hardening. These variables control the evolution of the yield surface and degradation of the elastic stiffness. The surface of typical fluency is shown on figure 3(a) in a deviator plane of tensions and on figure 3(b) in plane of tension.

These factors are closely related to the fracture energy dissipated to generate micro-cracks. The equations comprising the evolution of variables $\tilde{\varepsilon}_t^{pl}$ and $\tilde{\varepsilon}_c^{pl}$ are conveniently considered primarily the condition of uniaxial loading and then generalized to multiaxial conditions.

Briefly we have explained just some of the fundamental aspects of damage plasticity model in concrete, since the objective of this work is not to delve into the same; it is recommended the interested reader consult the references for a better understanding of this model.

Steel model

Based on studies of Bonilla et al. (2012) and Hernández et al. (2014), for the modelling of concrete-steel composite structures a bilinear behavior was adopted for the case of steel, based on Von Mises criterion, model also used by Rodríguez, Bonilla & Hernández (2012). This material behaves as a linear elastic reinforcement up to be reached yield stress (fys) and once exceeding this point behaves plastically. In code ABAQUS is used the *PLASTIC option for define the plastic region in model. In this case isotropic hardening to model yield surface was utilized.

Validation of the numerical modelling

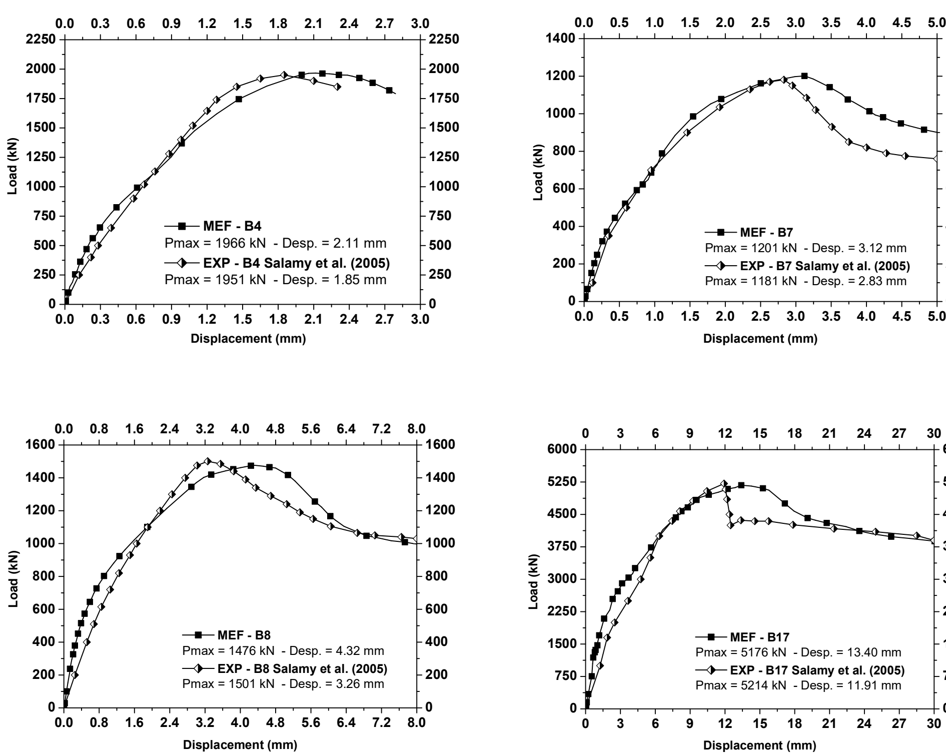
To validation the considerations taken, a numerical analysis is carried out on a group of beams specifically B4, B7, B8 and B17, that correspond to the experimental work presented by Salamy et al. (2005), taking ranges resistant capabilities that enable a parametric study to observe the influence of principal reinforcement, horizontal and vertical web reinforcement. Besides these beams are evaluated for different compressive concrete strength (f'$_c$) and for different ratio of shear span-effective depth (a/d).

Table 1. Comparison of the shear capacity of the numerical results with experimental. Source: Self-elaboration, 2015.

Reference	Beam	Experimental shear capacity $V_{EXP.}$ (kN)	Numerical shear capacity $V_{MEF.}$ (kN)	Dif. (%)	$\frac{V_{EXP.}}{V_{MEF.}}$
Salamy et al. (2005)	B4	975.50	983.15	+ 0.78	0.992
	B7	595.50	600.65	+ 0.86	0.991
	B8	750.50	738.24	- 1.63	1.017
	B17	2607.00	2588.22	- 0.72	1.007
Average:					1.002
Standard Deviation:					0.013
Coefficient of Variation (COV):					0.013

In Table 1 were observed a good correspondence of numerical modelling of the specimens with respect to experimental results, where in all cases are obtained errors less than 3%, that validate all the considerations and simplifications suggested for the process of modelling of reinforced concrete deep beams under static load with prevalence of shear.

Figure 4. Load vs displacement comparisons between numerical and experimental. Source: Self-elaboration, 2015.



It is important to stand out an appropriate correspondence in structural behavior of the beams, where the Figure 4 shown the similarities between of curves load vs displacement obtained numerically (MEF) with respect to the experimental curves (EXP) reported in a literature. It is further noted that the numerical curves show higher displacements with respect to experimental reference, however in the ascending limb is evidenced a better correspondence compared to the post peak branch where some differences may be inconsiderable if takes into account the precision obtained in estimating the ultimate shear strength.

Influence of the distribution of reinforcement on the ultimate shear capacity

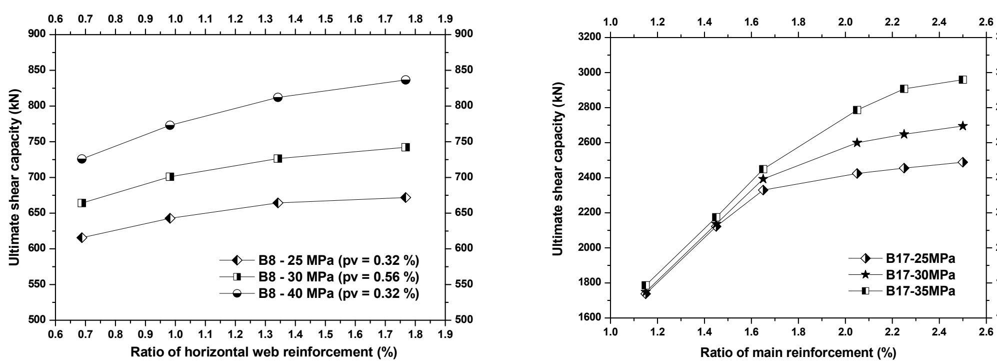
One of the main objectives of this paper is to perform a parametric study for evaluated of the contribution of each type of reinforcement by numerical simulation, to determine the influence of these reinforcements in terms of distribution and amount, given mainly by its demonstrated relationship with ultimate shear strength in reinforced concrete deep beams.

a) Influence of the main reinforcement

First is studied the influence of main reinforcement, wherein for a same type of reinforcement in the tie rods (number of bars) are assigned different diameters steel bars. It can be observed (see Figure 5) that in the beam (B4 - 25 MPa) the contribution of the main reinforcement is less significant than for beam (B4 - 35 MPa), showing that for lows compressive concrete strengths the failure occurs primarily in strut by the diagonal cracking, characteristic of shear.

In the comparing the beam (B8 - 35 MPa) with (B8 - 40 MPa), was observed a similarity in the behavior, where is visible the increased of ultimate shear capacity as increases the amount of main steel, it can be observed that a reinforcement with a ratio over 2.5% approximately, the curve tends to be asymptotic on the analyzed beams, showing a limit where the contribution of the reinforcement is less significant for geometric conditions and the materials properties examined.

Figure 5. Influence of main reinforcement in shear capacity (Beam B4, B8 and B17). Source: Self-elaboration, 2013.



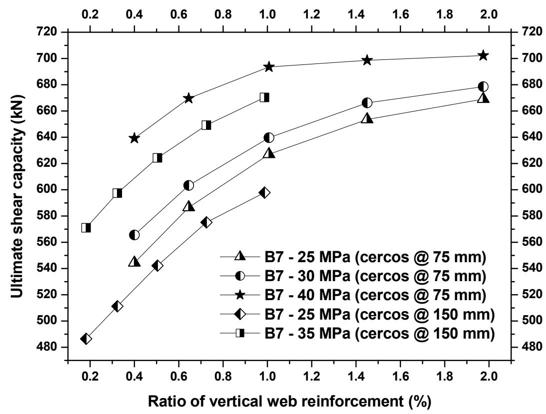
In the modelling of the beam (B17) is observed that for lower ratios of the main reinforcement (approximately less than 1.8%) and with different compressive strength of concrete are obtained similar shear capacity, clearly shows that the failure is dependent on the tie due to its low ratio, is also observed an increase in the shear capacity to greater ratio of main reinforcement are placed up to amounts where the curve tends to be asymptotical equally, showing that the fault can be caused by the strut, as it is sufficient the ratio of main reinforcement to resist traction force of the tie (for ratios more than 2.0%).

b) Influence of vertical web reinforcement

With the objective to evaluate the contribution of the vertical web reinforcement, in the Figure 6 shows the studies analyzed by numerical simulation of the beam (B7) for different compressive strength of concrete where the ratio of this type of reinforcement is increased.

The increase of ratio the vertical web reinforcement it is considered by assigning different areas to steel bars, where are modelling two trusses with different spacing between the stirrups, accomplish the evaluation of contribution on each beam and for different conditions on the model.

Figure 6. Influence of vertical web reinforcement in shear capacity (Beam B7). Source: Self-elaboration, 2013.



In the graph (Figure 6) can highlight some aspects to gain a better understanding of the phenomenon of failure that occurs in these structures. It is evident that as increases the amount of vertical reinforcement is increased ultimate shear capacity, but one can see that the curve tends to be asymptotic in all cases, thus showing a limit for beam where with materials tested and vertical reinforcement makes contribution an insignificant, that structurally and constructively would not have meaning of analysis.

If the beam B7- 25 MPa (stirrups @ 75 mm) is compared with the beam B7- 25 MPa (stirrups @ 150 mm), in terms of shear capacity can be seen as similar condition is that beam has or stirrups 6 mm @ 75 mm (pv = 0.40%) has greater shear capacity than the beam with stirrups 10 mm @ 150 mm (pv = 0.50%), from this behavior of the numerical models it could be stated that is better reinforce vertically with bars of small diameter spaced at short distance than the use of bars with greater diameters widely spaced (with the similar ratio of vertical reinforcement), in correspondence with experimental observations.

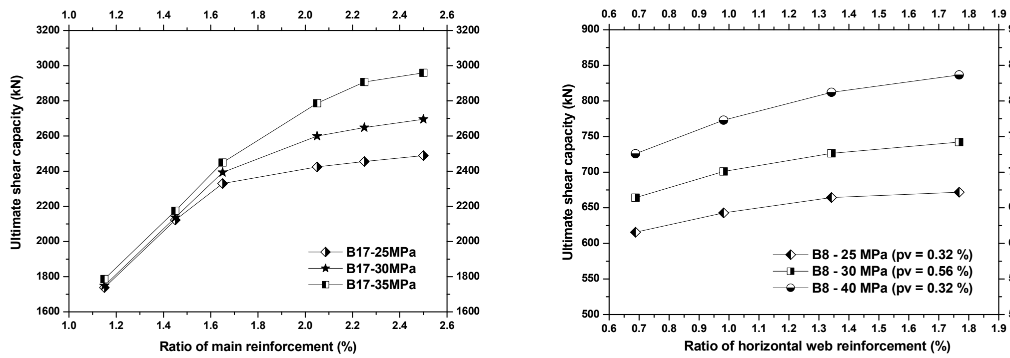
The comparative study shows that approximately for ratio of vertical reinforcement more than 1.2%, the ultimate shear capacity in this beam has less than 5% increases, so it would technically and constructively not meet objective placed of vertical reinforcement with high ratios. However, for smaller ratio to 1.0% of vertical reinforcement is observed a more significant increase in ultimate shear capacity.

c) Influence of horizontal web reinforcement

For the analysis of the influence of the horizontal web reinforcement was realized a numerical model on beam B8 and beam B17 for different distributions and ratios (Figure 7), where are distributed uniformly 2 to 8 horizontal bars on the effective depth of beam.

It can be seen as the behavior is similar in specimens, showing a moderate effect of the reinforcement on the shear capacity in these beams. The increased of shear capacity is not as significant with respect to the steel area placed. However, in the samples having the reinforcement ratio lower to 0.5%, it is notable load capacity increases with respect to specimen which do not have this type of reinforcement (this influence becomes more marked for beams with larger spacing between stirrups). Therefore, it is advisable to use this type of reinforcement by placing small diameters bars preferably with vertical reinforcement. Undoubtedly it has been found through numerical simulation that the use combined of vertical and horizontal reinforcement makes a greater contribution to the ultimate shear capacity, besides favoring the ductility in deep beams.

Figure 7. Influence of horizontal web reinforcement in shear capacity (Beam B8 and B17). Source: Self-elaboration, 2015.



Experimental data base

A total of 252 deep beams were taken from different consulted research in the international literature, taking the data collected in published researches such as: Sanad & Saka (2001), Tang & Tan (2004), Cladera & Marí (2005), Salamy et al. (2005), and Al-Gasham (2009). These studies helped in compiling a wide data base that allow the study of influence of principal factors and in some cases as reference test for numerical models. This database also used as a patron comparison to determine the accuracy in estimating the ultimate shear capacity of the proposed formulae in this paper. The distribution of main geometric parameters, materials and the reinforcement, taken for experimental database are shown on the Table 2, where the beams included were reported with shear failure.

Table 2. Summary of the characteristics and properties of the selected database beams. Source: Self-elaboration, 2015.

Authors	# beams		b_w (m)	d (m)	a/d	f'_c (MPa)	p_s (%)	p_{wh} (%)	p_{wv} (%)
Mau & Hsu (1989)	12	MIN	0.076	0.216	0.35	16.1	0.52	0.00	0.18
		MAX	0.102	0.724	2.08	24.6	1.94	1.01	2.45
Subedi, Vardi & Kubata (1986)	6	MIN	0.100	0.450	0.42	22.7	0.58	0.84	0.10
		MAX		0.850	1.53	34.7	1.16	1.42	0.24
Kong, Teng, Maimba, Tan & Guan (1994)	19	MIN	0.020	0.525	0.23	37.0	0.45	0.00	0.00
		MAX	0.150	0.940	1.71	76.0	1.93	1.04	0.70
Ramakrishnan & Ananthanarayana (1968)	10	MIN	0.066	0.349	0.27	12.5	0.14	0.00	0.00
		MAX	0.110	0.730	2.16	58.8	1.13	0.32	0.48
Smith & Vantsiotis (1982)	52	MIN	0.102	0.305	1.00	16.07	1.94	0.00	0.00
		MAX			2.08	22.68		0.91	1.25
Tan et al. (1995)	19	MIN	0.110	0.463	0.27	22.98	1.23	0.00	0.48
		MAX			2.70	58.84			
Kong et al. (1970)	31	MIN	0.076	0.216	0.35	18.55	0.50	0.00	0.00
		MAX		0.724	1.18	26.08	1.49	2.45	2.45
Cladera & Marí (2005)	18	MIN	0.195	0.260	2.77	26.0	1.47	0.00	0.11
		MAX	0.201	0.353	3.46	69.0	2.99		0.25
Salamy et al. (2005)	19	MIN	0.240	0.400	0.50	23.0	1.99	0.00	0.00
		MAX	0.840	1.400	1.50	37.8	2.11		0.80
Kong & Rangan (1998)	28	MIN	0.250	0.198	2.49	64.0	1.65	0.00	0.10
		MAX		0.297	3.30	89.0	4.46		0.26
(Zararis, 2003)	38	MIN	0.076	0.132	2.50	22.00	1.26	0.00	0.09
		MAX	0.457	1.200	4.00	125.00	4.54		0.78
TOTAL	252	MIN	0.020	0.132	0.23	12.50	0.14	0.00	0.00
		MAX	0.840	1.400	4.00	125.00	4.54	2.45	2.45

It must be noted that this database is also attached a total of 185 results of numerical simulation performed for a parametric study, feature which contributes significantly to a better correlation and adjustment of the factors that take into account the direct influence of ratios for reinforcement used on deep beams.

Results

Estimation of the ultimate shear capacity

Numerous studies show that the hypothesis of the flat sections does not comply in these types of structures, as stress fields are not distributed linearly, even in the elastic range, so that the conventional methods cannot be applied to calculate of strain and section properties. Consistent with this have been adopted various procedures international, some with a completely empirical foundation and others with a more analytical approach.

In this paper are proposed two formulations for estimating the ultimate shear capacity in deep beams, from empirical models derived from nonlinear regression analysis with the help of statistical software SPSS v-11.5.1 (2002). We study various formats of statistical forecasting models, being selected those that best predict the shear capacity in this type of beams. Finally, an adequate estimates of the regression models are obtained, in correspondence with the experimental results and improved the prediction compared to the other methods studied, published in the literature consulted.

The analysis of the experimental database taken a first formulation is proposed, which has a coefficient of determination R^2 of 0.915 for 102 beams selected, appreciated that in structure appear the nine parameters, where their influence on the ultimate shear capacity has been demonstrated in actual tests.

$$V = \left(\frac{2600 \cdot b_w \cdot d^{\,1.55} \cdot f_c'^{\,0.10}}{a^{\,0.65}} \right) + 0.015 \cdot f_{y-med} \cdot \left(0.8 \cdot \rho_{wh} + 2 \cdot \rho_{wv} \right) + 0.035 \cdot f_{y-s} \cdot \rho_s \tag{4}$$

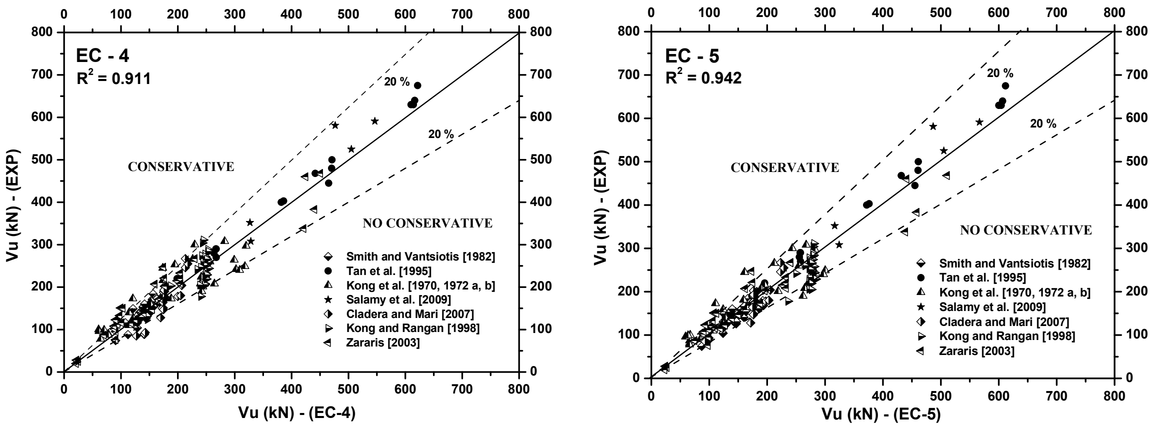
To validate the accuracy in estimating the ultimate shear capacity of the first formulation (EC-4), were analyzed a set of 100 beams of other test that were not taken for the conception of the model, showing an adequate estimate a coefficient of determination R^2 of 0.856. It is proposed a second formulation where involved these nine parameters, this has a coefficient of determination R^2 of 0.945 for the same database previously taken.

$$V = \left(\frac{240 \cdot b_w \cdot d^{\,1.65} \cdot f_c'^{\,0.10}}{a^{\,0.65}} \right) \cdot \left[f_{y-med} \cdot \left(0.5 \cdot \rho_{wh} + 1.3 \cdot \rho_{wv} \right) + 0.85 \cdot f_{y-s} \cdot \rho_s \right]^{0.4} \tag{5}$$

The validation of the accuracy of the proposed model (EC-5) give values of ultimate shear capacity very suitable in correspondence with the experimental and numerical results of the validation, maintaining a high coefficient of determination R^2 of 0.882 in the adjustment.

To assess the accuracy of each method are takes into account some statistical variables of interest that enable a better assessment, if we compare the average between VEXP /VTEOR have that for EC-4 is 0.994 and for EC-5 is 1.020, the standard deviation for EC-4 is 0.196 and for EC-5 is 0.182, statistically are observed superiority of the second formulation which is reflected graphically to some extent (see Figure 8). Finally, it is necessary to mention that even though the first model (EC-4) has a coefficient of determination at the lowest setting, this model has practical and engineering sense, in accordance with traditional procedures, since it clearly distinguishes the individual contribution of concrete and reinforcing steel.

Figure 8. Validation of the proposed formulations: a) First model (EC-4); b) Second model (EC-5). Source: Self-elaboration, 2015.



Comparison between the proposed formulations and international methods

A comparative study was performed to determine the accuracy in estimating the ultimate shear capacity, which analyses also some methods implemented in international standards such as the approach strut and tie of ACI Comitee 318 (2011) (Appendix A), procedures of European Standard EN 1992-1-1 (2004) and the method proposed by Tang & Tan (2004). These estimates of international methods are compared with the predictions of the empirical formulations proposed, using as a standard for comparison 50 beams tested in the studies of Smith & Vantsiotis (1982), since it has all the necessary tested parameters for the implementation of all methods analyzed.

The results from the analysis are summarized in Table 3, where the precision of the consulted international methods with the empirical procedures proposed in this paper is compared. The numerical values from the relationship: VEXP / Vn-Tang (2004) were not calculated by the authors of this work, as they were taken directly from the study of Tang & Tan (2004).

Table 3. Comparison of the methods in the beams tested by (Smith & Vantsiotis, 1982). Source: Self-elaboration, 2015.

Beam	a (m)	a/d	f'_c (MPa)	ρ_s (%)	ρ_{wh} (%)	ρ_{wv} (%)	V_{EXP} (kN)	V_{EXP} / $V_{n\text{-Tang (2004)}}$	V_{EXP}/ $V_{n\text{-(ACI-11)}}$	V_{EXP}/ $V_{n\text{-(EC2)}}$	V_{EXP}/ (EC-4)	V_{EXP}/ (EC-5)
0A0-44	0.305	1.00	20.48	1.94	0.00	0.00	140.0	1.129	1.448	1.496	0.918	1.003
0A0-48	0.305	1.00	20.93	1.94	0.00	0.00	136.0	1.088	1.376	1.425	0.890	0.972
1A1-10	0.305	1.00	18.69	1.94	0.23	0.28	161.0	1.376	1.459	1.663	1.025	1.040
1A3-11	0.305	1.00	18.03	1.94	0.45	0.28	149.0	1.263	1.400	1.590	0.942	0.945
1A4-12	0.305	1.00	16.07	1.94	0.68	0.28	141.0	1.294	1.487	1.674	0.890	0.885
1A4-51	0.305	1.00	20.55	1.94	0.68	0.28	171.0	1.315	1.410	1.619	1.059	1.047
1A6-37	0.305	1.00	21.06	1.94	0.91	0.28	184.0	1.373	1.480	1.704	1.126	1.101
2A1-38	0.305	1.00	21.68	1.94	0.23	0.63	175.0	1.287	1.368	1.578	1.067	1.024
2A3-39	0.305	1.00	19.75	1.94	0.45	0.63	171.0	1.326	1.467	1.679	1.040	0.992
2A4-40	0.305	1.00	20.34	1.94	0.68	0.63	172.0	1.303	1.432	1.644	1.033	0.977
2A6-41	0.305	1.00	19.13	1.94	0.91	0.63	162.0	1.266	1.435	1.638	0.968	0.910
3A1-42	0.305	1.00	18.41	1.94	0.23	1.25	161.0	1.288	1.482	1.686	0.942	0.853
3A3-43	0.305	1.00	19.24	1.94	0.45	1.25	173.0	1.331	1.523	1.740	1.000	0.900
3A4-45	0.305	1.00	20.82	1.94	0.68	1.25	179.0	1.288	1.457	1.675	1.019	0.912
3A6-46	0.305	1.00	19.93	1.94	0.91	1.25	168.0	1.235	1.428	1.636	0.950	0.848
0B0-49	0.368	1.21	21.68	1.94	0.00	0.00	149.0	1.307	1.634	1.512	1.073	1.199
1B1-01	0.368	1.21	22.06	1.94	0.23	0.24	148.0	1.244	1.276	1.337	1.026	1.074
1B3-29	0.368	1.21	20.10	1.94	0.45	0.24	144.0	1.286	1.363	1.416	0.994	1.031
1B4-30	0.368	1.21	20.82	1.94	0.68	0.24	141.0	1.216	1.288	1.343	0.960	0.984
1B6-31	0.368	1.21	19.51	1.94	0.91	0.24	154.0	1.375	1.501	1.556	1.042	1.059
2B1-05	0.368	1.21	19.17	1.94	0.23	0.42	129.0	1.183	1.280	1.325	0.888	0.906
2B3-06	0.368	1.21	19.00	1.94	0.45	0.42	131.0	1.202	1.312	1.356	0.892	0.902

2B4-07	0.368	1.21	17.48	1.94	0.68	0.42	126.0	1.223	1.371	1.408	0.854	0.858
2B4-52	0.368	1.21	21.79	1.94	0.68	0.42	150.0	1.230	1.309	1.370	1.000	0.999
2B6-32	0.368	1.21	19.75	1.94	0.91	0.42	145.0	1.261	1.397	1.449	0.963	0.957
3B1-08	0.368	1.21	16.24	1.94	0.23	0.63	131.0	1.351	1.534	1.568	0.894	0.891
3B1-36	0.368	1.21	20.41	1.94	0.23	0.77	159.0	1.359	1.482	1.542	1.052	1.027
3B3-33	0.368	1.21	19.00	1.94	0.45	0.77	159.0	1.420	1.591	1.646	1.047	1.017
3B4-34	0.368	1.21	19.24	1.94	0.68	0.77	155.0	1.360	1.533	1.586	1.008	0.974
3B6-35	0.368	1.21	20.65	1.94	0.91	0.77	166.0	1.372	1.529	1.592	1.063	1.019
4B1-09	0.368	1.21	17.10	1.94	0.23	1.25	154.0	1.453	1.713	1.757	0.986	0.928
0C0-50	0.457	1.50	20.69	1.94	0.00	0.00	116.0	1.234	1.568	1.228	0.935	1.080
1C1-14	0.457	1.50	19.24	1.94	0.23	0.18	119.0	1.293	1.730	1.296	0.932	1.024
1C3-02	0.457	1.50	21.89	1.94	0.45	0.18	124.0	1.216	1.574	1.201	0.950	1.029
1C4-15	0.457	1.50	22.68	1.94	0.68	0.18	131.0	1.236	1.144	1.228	0.989	1.059
1C6-16	0.457	1.50	21.79	1.94	0.91	0.18	123.0	1.183	1.222	1.196	0.919	0.976
2C1-17	0.457	1.50	19.86	1.94	0.23	0.31	124.0	1.292	1.397	1.312	0.955	1.027
2C3-03	0.457	1.50	19.24	1.94	0.45	0.31	104.0	1.106	1.523	1.132	0.793	0.846
2C3-27	0.457	1.50	19.31	1.94	0.45	0.31	116.0	1.234	1.425	1.259	0.884	0.943
2C4-18	0.457	1.50	20.44	1.94	0.68	0.31	125.0	1.263	1.357	1.288	0.937	0.989
2C6-19	0.457	1.50	20.75	1.94	0.91	0.31	124.0	1.228	1.121	1.260	0.917	0.960
3C1-20	0.457	1.50	21.03	1.94	0.23	0.56	141.0	1.369	1.234	1.416	1.052	1.093
3C3-21	0.457	1.50	16.55	1.94	0.45	0.56	125.0	1.471	1.690	1.564	0.937	0.974
3C4-22	0.457	1.50	18.27	1.94	0.68	0.56	128.0	1.376	1.519	1.462	0.941	0.969
3C6-23	0.457	1.50	19.00	1.94	0.91	0.56	137.0	1.412	1.661	1.509	0.993	1.015
4C1-24	0.457	1.50	19.58	1.94	0.23	0.77	147.0	1.485	1.429	1.575	1.078	1.097
4C3-04	0.457	1.50	18.55	1.94	0.45	0.63	129.0	1.372	1.544	1.453	0.952	0.979
4C3-28	0.457	1.50	19.24	1.94	0.45	0.77	153.0	1.561	1.593	1.666	1.110	1.125
4C4-25	0.457	1.50	18.51	1.94	0.68	0.77	153.0	1.611	1.777	1.726	1.100	1.111
4C6-26	0.457	1.50	21.24	1.94	0.91	0.77	137.0	1.509	1.661	1.592	1.126	1.128
							Average:	1.315	1.459	1.491	0.987	0.993
All beams: b_w = 0.102 m; d = 0.305 m; f_{ys} = 431.0 MPa; f_{yw} = 483.5 MPa.							Deviation:	0.106	0.145	0.169	0.075	0.080
							COV:	0.081	0.099	0.113	0.076	0.081

At the bottom of the Table 3 shows some statistical data demonstrating the superiority in forecasting models proposed regarding the procedures studied are given. Here we find that the average value between VEXP / VTEOR is 0.982 with a COV of 0.076 for the EC-4 and average value 0.993 with a COV of 0.081 for the EC-5, certainly are achieved high accuracy in estimating the ultimate shear capacity in the beams tested in studies of (Smith & Vantsiotis, 1982).

Conclusions

Accurate nonlinear finite element models have been developed to investigate the behavior of reinforced concrete deep beams under static load with prevalence of shear. From the precision obtained in the calibration of models, a parametric study of the influence of types of reinforcement was conducted, for obtaining a better correlation of these factors in the proposed formulations. In the parametric study it was observed that:

- It is observed that with increasing the amount of main reinforcement in the beams shear capacity increases, reaching a limit where the increase of this amount is not significant because failure is determined by the behavior of the strut.
- Increasing the amount of vertical reinforcement increases the shear capacity, reaching a limit where their contribution is insignificant. It is recommended stirrups with small diameter (less than 10 mm), placing them closer spacing up to amounts close to 1%.
- The increase in the amount of horizontal web reinforcement has a moderate effect on the shear capacity; however, a favorable effect even at low amounts observed when reinforcement is placed in combination with the vertical web reinforcement contributing to the work of the stirrups and also has a favorable effect on the ductility of the failure.

In this work two simple and practical empirical formulations are also proposed, which provides more accurate results compared to studied international procedures, appreciating an underestimation by the latters, in some cases excessive, of the shear capacity in reinforced concrete deep beams with prevalence of shear.

References

ABAQUS. (2000). ABAQUS/CAE User's Manual. Hibbitt, Karlsson & Sorensen, Inc.

ACI Comitee 318. (2011b). Building Code Requirements for Structural Concrete (ACI 318-11) and Commentary. American Concrete Institute.

Adhikary, S. Das, Li, B., & Fujikake, K. (2013). Strength and behavior in shear of reinforced concrete deep beams under dynamic loading conditions. *Nuclear Engineering and Design*, *259*(ST5), 14–28. https://doi.org/10.1016/j.nucengdes.2013.02.016

Al-Gasham, T. S. (2009). Treatment of Shear of Reinforced Concrete Slender Beams with Web Reinforcement in well-known Structural International Codes. *Al-Qadisiya Journal For Engineering Sciences*, *2*(2), 215–228.

Bonilla, J., Bezerra, L. M., Larrúa, R., Recarey, C., & Mirambell, E. (2015). Modelación numérica con validación experimental aplicada al estudio del comportamiento de conectores tipo perno de estructuras compuestas de hormigón y acero. *Revista Ingeniería de Construcción*, *30*(1), 53–68. https://doi.org/10.4067/S0718-50732015000100005

Bonilla, J., Mirambell, E., Larrúa, R., & Recarey, C. (2012). Behavior and strength of welded stud shear connectors in composite beam. *Revista Facultad Ingeniería Universidad de Antioquia*, *63*, 93–104.

Cladera, A., & Marí, A. R. (2005). Experimental study on high-strength concrete beams failing in shear. *Engineering Structures*, *27*(10), 1519–1527. https://doi.org/10.1016/j.engstruct.2005.04.010

De Paiva, H. A. R., & Siess, C. P. (1965). Strength and Behaviour of Deep Beams in Shear. *Journal of Structural Engineering*, *91*(ST5), 19–41.

Ellobody, E., & Young, B. (2006). Performance of shear connection in composite beams with profiled steel sheeting. *Journal of Constructional Steel Research*, *62*(7), 682–694. https://doi.org/10.1016/j.jcsr.2005.11.004

European Standard EN 1992-1-1. (2004). Eurocode 2: Design of concrete structures, Part 1: General rules and rules for buildings (2004). The European Union.

Hernández, H., Bonilla, J., & Rodríguez, G. (2014). Estudio del comportamiento de vigas compuestas de hormigón y acero mediante simulación numérica. *Revista Ingeniería de Construcción*, *29*(1), 5–21. https://doi.org/10.4067/S0718-50732014000100001

Kong, F. K., Robins, P. J., & Cole, D. F. (1970). Web Reinforcement Effects on Deep Beams. *ACI Journal*, *67*(12), 1010–1018.

Kong, F. K., Teng, S., Maimba, P., Tan, K. H., & Guan, L. W. (1994). Single-Span, Continuous, and Slender Deep Beams Made of High-Strength Concrete. *ACI Special Publication*, *149*, 413–432.

Kong, P. Y., & Rangan, B. V. (1998). Shear Strength of High-Performance Concrete Beams. *ACI Structural Journal*, *94*(6), 677–688.

Lee, J., & Fenves, G. L. (1998). Plastic-Damage Model for Cyclic Loading of Concrete Structures. *Journal of Engineering Mechanics*, *124*(8), 892–900. https://doi.org/10.1061/(ASCE)0733-9399(1998)124:8(892)

Lubliner, J., Oliver, J., Oller, S., & Oñate, E. (1989). A Plastic-Damage Model for Concrete. *International Journal of Solids and Structures*, *25*(3), 229–326. https://doi.org/10.1016/0020-7683(89)90050-4

Mau, S. T., & Hsu, C. T. (1989). Formula for the Shear Strength of deep Beams. *ACI Structural Journal*, *86*(8), 516–523.

Norambuena-Contreras, J., Castro-Fresno, D., Del Coz, J. J., & García, P. J. (2011). Simulación numérica de una mezcla asfáltica usando MEF y diseño de experimentos. *Revista de La Construcción*, *10*(2), 4–15. https://doi.org/10.4067/S0718-915X2011000200002

Ramakrishnan, V., & Ananthanarayana, Y. (1968). Ultimate Strength of Deep Beams in Shear. *ACI Journal*, *65*, 87–98.

Rodríguez, G., Bonilla, J., & Hernández, J. J. (2012). Aplicación de la simulación numérica al estudio del comportamiento de vigas de gran peralto de hormigón armado. *Revista Ingeniería Civil*, *167*, 101–116.

Sagaseta, J., & Vollum, R. L. (2010). Shear design of short-span beams. *Magazine of Concrete Research*, *62*(4), 267–282. https://doi.org/10.1680/macr.2010.62.4.267

Salamy, M. R., Kobayashi, H., & Unjoh, S. (2005). Experimental and analytical study on RC deep beams. *Asian Journal of Civil Engineering*, *6*(5), 487–499.

Sanad, A., & Saka, M. P. (2001). Prediction of Ultimate Shear Strength of Reinforced Concrete Deep Beams using Neuronal Networks. *Journal of Structural Engineering*, *127*(7), 818–828.

Smith, K., & Vantsiotis, A. (1982). Shear strength of deep beams. *ACI Journal*, *79*(3), 201–213.

Subedi, N. K., Vardy, A. E., & Kubotat, N. (1986). Reinforced concrete deep beams some test results. *Magazine of Concrete Research*, *38*(137), 206–219. https://doi.org/10.1680/macr.1986.38.137.206

Tan, K. H., Kong, F. K., Teng, S., & Guan, S. (1995). High-Strength Concrete Deep Beams With Effective Span and Shear Span Variations. *ACI Structural Journal*, *92*(4), 395–405.

Tang, C. Y., & Tan, K. H. (2004). Interactive Mechanical Model for Shear Strength of Deep Beams. *Journal of Structural Engineering*, *130*(10), 1534–1544. https://doi.org/10.1061/(ASCE)0733-9445(2004)130:10(1534)

Zararis, P. D. (2003). Shear Strength and Minimum Shear Reinforcement of Reinforced Concrete Slender Beams. *ACI Structural Journal*, *100*(2), 203–214.